TIDAL WAVES – THE YRAST STATES IN NEAR-SPHERICAL NUCLEI

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Abstract

The new concept of nuclear tidal waves allows one to calculate the sequence of states with maximal angular momentum (yrast states). These states correspond to a surface wave running around the nucleus as tidal waves run over the earth's oceans. Such a wave corresponds to a static deformation in the rotating frame of reference, which allows one to calculate its properties by means of the microscopic cranking model. There exists a large group of transitional nuclei that show a behavior between harmonic vibration and rotation, a description of which has been a long-standing challenge to nuclear theory. The new concept of nuclear tidal waves allows one to calculate the sequence of states with maximal angular momentum (yrast states). These states correspond to a surface wave running around the nucleus as tidal waves run over the earth's oceans. Such a wave corresponds to a static deformation in the rotating frame of reference, which allows one to calculate its properties by means of the microscopic cranking model.

Introduction

The appearance of triaxial nuclear shapes and the question how to prove their existence is a long-standing problem in nuclear physics. The recent discovery of wobbling rotational bands is considered as first clear cut evidence for this type of nuclear shape. So far wobbling bands have only be found in the Lu isotopes. In this paper, the results of experimental search for wobbling bands in 163Tm are theoretically analyzed. Calculations of the energy and electromagnetic transition probabilities are carried out in the framework of the microscopic Tilted Axis Cranking (TAC) model. The calculations account well for the data. It is found that the bands have a triaxial strong deformation (TSD).

The calculations account well for the data. It is found that the bands have a triaxial strong deformation (TSD). However, the two observed rotational sequences do not correspond to a zero and one wobbling phonon but to the two signature branches of the odd proton orbital, which is a very low lying particle-hole excitation. It is demonstrated that in general wobbling phonons compete with particle-hole excitations. With present-day types of experiments, the two modes can only be disentangled if the wobbling states are the lowest excitations, as in the Lu isotopes, whereas in many other cases, including163Tm, the particle-hole excitations make up th espectrum of observed rotational TSD bands.

The nucleus consisting of many nucleons is a quantum-mechanical many-body system. Quantum Mechanics is the fundamental theory to describe the nuclear structure, and the Schroedinger equation is the basic equation to describe the nuclear behavior. However it is almost impossible to exactly solve this problem for nuclei. For such a complicated system we have to introduce simple models which are tractable and provide insight. Let me review some models that are relevant to my paper.

For the first time such calculations are carried out in this thesis for nuclei with Z= 44, 46, 48 and N= 56-66. The calculations reproduce very well energies and E2-transitions probabilities of these nuclei. The low-spin parts show the expected gradual transition from vibration-like behavior of the nuclides near the closed shell (Z=48,N=56) to rotation-like behavior of the nuclide farthest in the open shell (Z= 44,N=66). In addition, the strong irregularity (back-bending) caused by the alignment of the angular momentum of two h11=2 neutrons with the rotational axis is well accounted for.

At spins larger than 10~h, where the back-bending occurs; all nuclei behave more rotation-like, which is borne out by the calculations in accordance with experiment. Such a detailed theoretical description of the yrast states of transitional nuclei has been achieved for the first time.

Theoretical Models

Liquid Drop Model

The nuclear size is smaller than 2×10−14 m. Some nuclei are spherical. However, many of them are not exactly spherical but slightly deformed or strongly deformed. The charge distribution of a spherical nucleus is well represented by the function

$\rho(r) = \rho 0\ (1+e(r-R)/a)$

r = distance from the center, which is called the Woods-Saxon potential [1]. The parameters ρ0, R and are adjusted to the experimental charge distribution measured by electron scattering. The parameter R is the radius of the nucleus, and the parameter determines the surface thickness. The nuclear radius is given by:

$R = 1.2 \times A1/3$ fm

Where4 A = N + Z, and N is the number of neutrons and Z the number of protons. The volume of a nucleus, is

$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 1.2^3 \times A$

And its density is

$\rho = AV \approx 0.138\ fm^{-3}$

The nuclear density is almost the same for all nuclei. This is called the saturation of nuclear density .

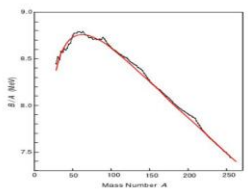

Figure 1.1. Comparison of the binding energies between semi-empirical mass formula and experiment. The points indicate the experimental data of the binding energy per nucleon. The curve shows the values obtained by the Weizsaecker-Bethe mass formula.

Spherical Shell Model

The original idea of the nuclear shell model [3] is analogous with the atomic shell model. However, there are substantial differences between atoms and nuclei. Since the nucleus is like a liquid drop, the potential created by the nucleons must be almost constant inside of the nucleus and change to zero in a thin surface. However this potential does not yet reproduce the magic numbers.

M.G. Mayer and H.D. Jensen independently solved the problem by proposing an average potential, which included a strong spin-orbit term (See Fig. 2). The modern version is the spherical Woods-Saxon potential which reproduces the nuclear magic numbers very well. The shell structure of nuclei is one of the basic concepts of the nuclear-structure theory.

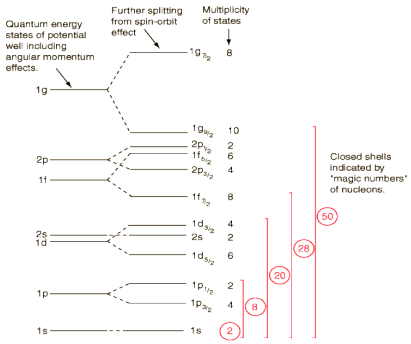

Figure 2. Nuclear Shell Model

Nuclear Collective Model

In addition to the motion of the individual nucleus inside the nuclear potential, the potential itself, rotates and vibrates, which is called collective motion. The collective model describes these excitation modes.

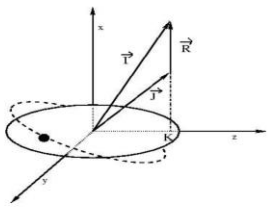

Figure 1.6. Schematic of the coupling of the collective angular momentum

Where $\Im$ is the moment of inertia. In the quantum mechanics, it will turn into

$$E = \frac{\hbar^2}{2\Im} I(I+1)$$

For a band with a non-zero projection of J onto the symmetry axis, denoted by K, the energy becomes

$$E = \frac{\hbar^2}{2\Im^{(0)}}[I(I+1) - K^2]$$

If the nucleus rotates very rapidly, the valence particles may contribute an amount of angular momentum i (sometimes called also ix) along the axis perpendicular to the symmetry axis, which is called aligned angular momentum or alignment (see Fig. 1.7). In this case the energy is

$$E = \frac{\hbar^2}{2\Im_0}(I + 1/2 - i)^2$$

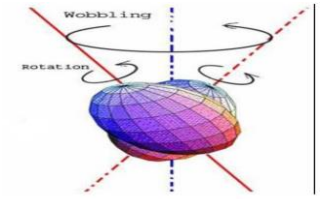

Figure 1.7. Nuclear wobbling motion

Goal of this paper

At present, the structure of triaxial strongly deformed nuclei is in the focus of nuclear physics. Nuclei with N~94 and Z~71 [7] provide a possibility to study strongly deformed nuclear shape with triaxiality. The wobbling mode is uniquely related to the triaxiality. Evidence of the existence of the wobbling mode has been found and identified in 163Lu[8] and its isotopes,165Lu and 167Lu[9], which encouraged people to consider wobbling as a general phenomenon in A~163 region.

However, recently two interacting strongly-deformed bands in 163Tm have been identified by Prof. U. Garg's group [10] who did not find evidence for the presence of the wobbling mode. In order to resolve this discrepancy, we investigated the nuclei in this region, in order to explain the properties of new found bands and of other strongly deformed bands in Tm-Lu-Hf-Yb isotopes.

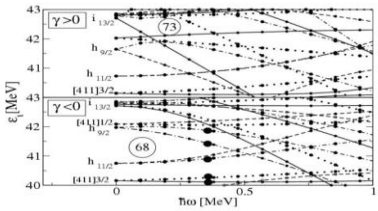

Figure 3.11. Single-proton routhians as function of rotational frequency in TSD minima 1 (top panel) and 2 (bottom panel). The line type indicates by (π,α)=(+,1/2) full;(+,-1/2) dot;(-,1/2) dash;(-,-1/2) dash dot. Occupied levels are labelled by large filled circles.

On the other hand, one of the major themes in nuclear structure research is shape evolution with nucleon number and angular momentum. Collective excitations of nuclei have two types of modes, one is vibration and another one is rotation. In A~110 region, a transition from the vibrational motion at low spin to rotational motion at high spin has been suggested. The microscopic description of anharmonic vibrational motion has been a big challenge to theory. For the first time we applied the cranking method to interprete the behavior of surface vibrational nuclei, which is a crucial step in meeting this challenge.

Acknowledgement

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